Design Parameters of a Benchtop Rig Hydraulics Similitude

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Introduction

During drilling operations, formation pore pressure and fracture gradients create a narrow window within which the equivalent circulation density (ECD) of the drilling fluid must fall in. Otherwise, undesirable events such as formation damage, kicks, lost circulation, and wellbore breakout could take place. Therefore, correct measurement of the frictional pressure losses and accurate calculation of ECD are important in drilling engineering design. Frictional pressure losses along the wellbore annulus are the key element for finding the equivalent circulating density of the circulating fluid.

Objective

- Perform dimensional analysis on the involved variables, dimensions, and dimensionless groups for the wellbore annular flow of power-law fluids with cuttings.
- Obtain scaling relations between laboratory's fluid flow rate, solid particles flow rate, and inner pipe rotation in terms of those in the wellbore.
- Use the obtained relations to find the frictional pressure loss of the wellbore in terms of that of the laboratory flow-loop device

Methods

Table 1: Dimensional Analysis Breakdown of the two-phase annular flow of the wellbore

Group		Quantity	Expression	
Involved variables	Fluid	m = 9	$\Delta p/\Delta x, \rho_{fl}, R_o, R_i, e, \omega, U, K_{fl}, n$	
Involved variables	Solid particles	n = 3	D_s, ρ_s, ϕ	
Involved dimensions		3	L, M, T	
Dimensionless variables	Geometric	4	Fluid	$\frac{R_i}{D_H}, \frac{e}{D_H}$
			Solid Particles	$\frac{D_s}{D_H}, \phi$
	Dynamic	4	Fluid	$Re = \frac{\rho U D_H}{\mu_e}$, $Ta = \frac{\rho \omega R_i D_H}{\mu_e}$, $f = \frac{D_H}{4\rho U^2}\frac{\Delta p}{\Delta x}$
			Solid Particles	$Re_s = \frac{\rho_{fl} V_{sl} D_s}{\mu_{fl}}$
	Rheological	1	Fluid	n

To ensure full similitude, the dimensionless groups in Table 1 must be equal between the the model scale and prototype scale. Which could be achieved by imposing the following:

Geometric similarity:

$$\psi = \frac{R_{i,M}}{R_{i,P}} = \frac{R_{o,M}}{R_{o,P}} = \frac{D_{H,M}}{D_{H,P}} = \frac{e_M}{e_P}$$

$$\phi_M = \Phi_P$$

Kinematic similarity:

$$\eta = \frac{\omega_M R_{i,M}}{U_M} = \frac{\omega_P R_{i,P}}{U_P}$$

Dynamic similarity:

$$Re_{mix,M} = Re_{mix,P} \rightarrow \frac{\rho_M U_M D_{H,M}}{\mu_{a,mix,M}} = \frac{\rho_P U_P D_{H,P}}{\mu_{a,mix,P}}$$

$$Ta_M = Ta_P \rightarrow \frac{\rho_M \omega_M R_{i,M} D_{H,M}}{\mu_{a,mix,M}} = \frac{\rho_P \omega_P R_{i,P} D_{H,P}}{\mu_{a,mix,P}}$$

$$Re_{P,M} = Re_{P,P} \rightarrow \frac{\rho_{fl,M} V_{sl,M} D_{s,M}}{\mu_{a,fl,M}} = \frac{\rho_{fl,P} V_{sl,P} D_{s,P}}{\mu_{a,fl,P}}$$

Rheological similarity:

$$n_M = n_P$$

Once all the above similarities are achieved, the dimensionless frictional factor of the model will be equal to the prototype:

$$\frac{D_{H,M}}{4\rho_M U_M^2}\left(\frac{\Delta p}{\Delta x}\right)_M = \frac{D_{H,P}}{4\rho_P U_P^2}\left(\frac{\Delta p}{\Delta x}\right)_P$$

Results

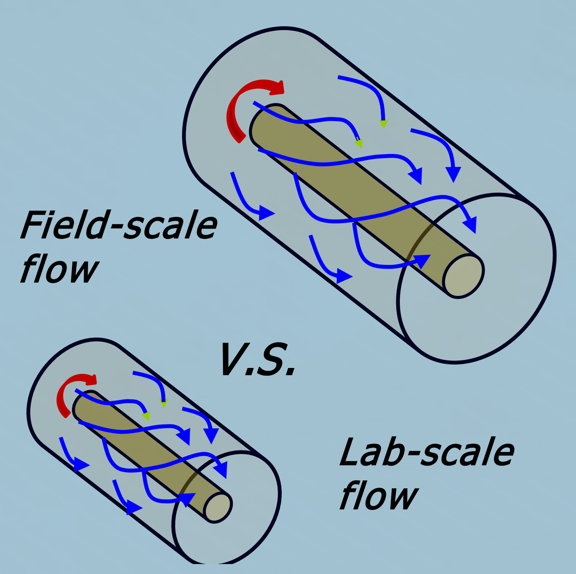
Upon simplification, the following closed-form formulas was obtained as a scale-relations:

$$Q_{fl,M} = -A_P \psi^2 V_{sl,M}(\phi - 1)\phi + \left\{\frac{\rho_{mix,P}}{\rho_{mix,M}}\frac{K_{fl,M}}{K_{fl,P}}\right\}^{\frac{1}{2-n}} \left(Q_{fl,P} + A_P V_{sl,P}(\phi-1)\phi\right)\psi^{\frac{4-3n}{2-n}}$$

$$Q_{s,M} = A_p \psi^2 V_{sl,M}(\phi - 1)\phi + \left\{\frac{\rho_{mix,P}}{\rho_{mix,M}}\frac{K_{fl,M}}{K_{fl,P}}\right\}^{\frac{1}{2-n}} \left(Q_{s,P} - A_P V_{sl,P}(\phi-1)\phi\right)\psi^{\frac{4-3n}{2-n}}$$

$$\omega_M = \omega_P \left(\frac{\rho_{mix,P}}{\rho_{mix,M}}\ \frac{K_{fl,M}}{K_{fl,P}}\right)^{\frac{1}{2-n}} \psi^{\frac{-2}{2-n}}$$

$$\frac{\Delta p_P}{\Delta x_P} = \psi^{\frac{n+2}{2-n}}\left(\frac{\rho_P}{\rho_M}\right)^{\frac{n}{2-n}}\left(\frac{K_{fl,M}}{K_{fl,P}}\right)^{\frac{2}{n-2}}\frac{\Delta p_M}{\Delta x_M}$$



Where the slip velocity definition for power law fluids is:

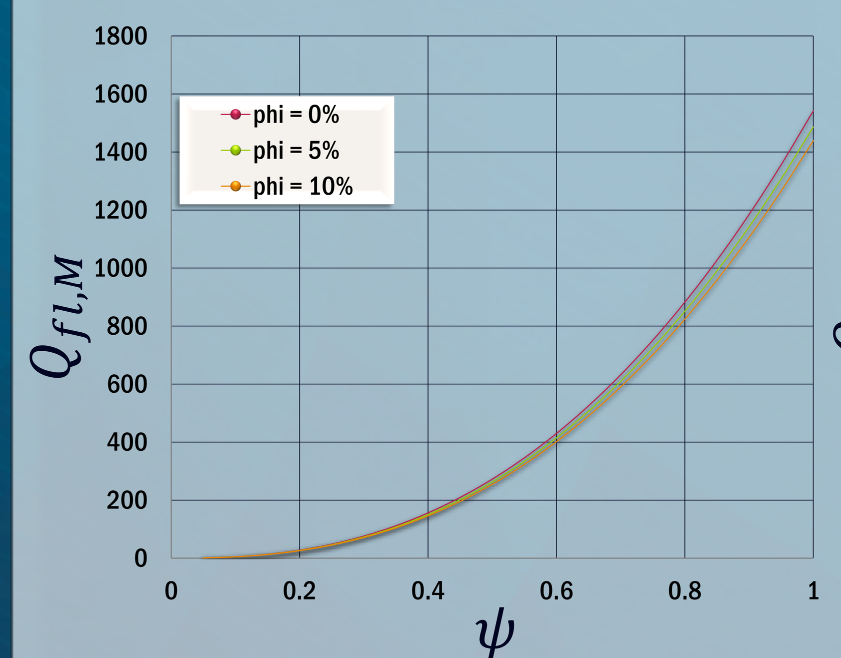
$$V_{sl} = \left[\frac{4}{3} g \frac{[(D_S)^{1+vn}](\rho_s - \rho_{fl})}{a F_s K^v \rho_{fl}^{1-v}}\right]^{(1/[2-v(2-n)]}$$

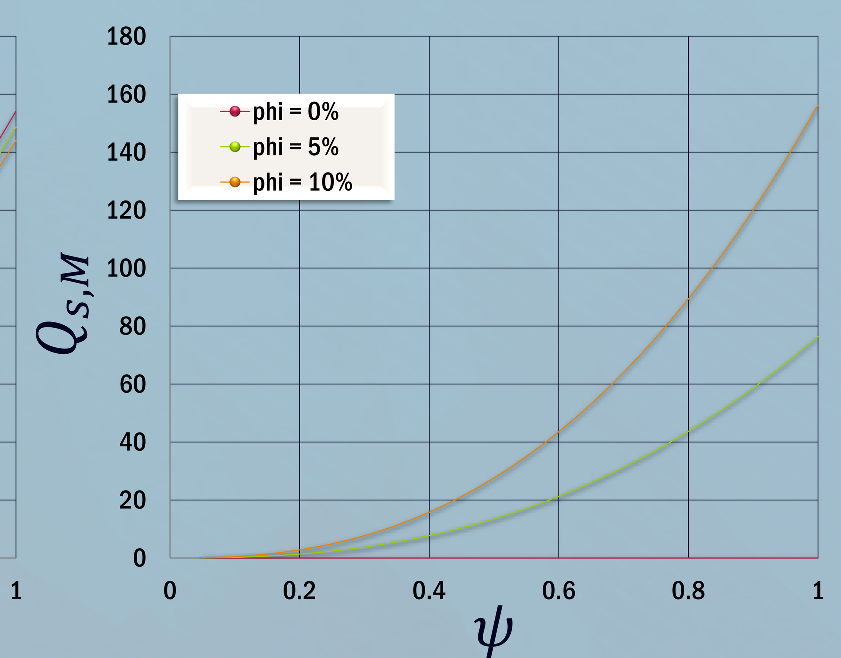
Laminar flow: $a = 39.8 - 9n$, $v = 1.2 - 0.47n$

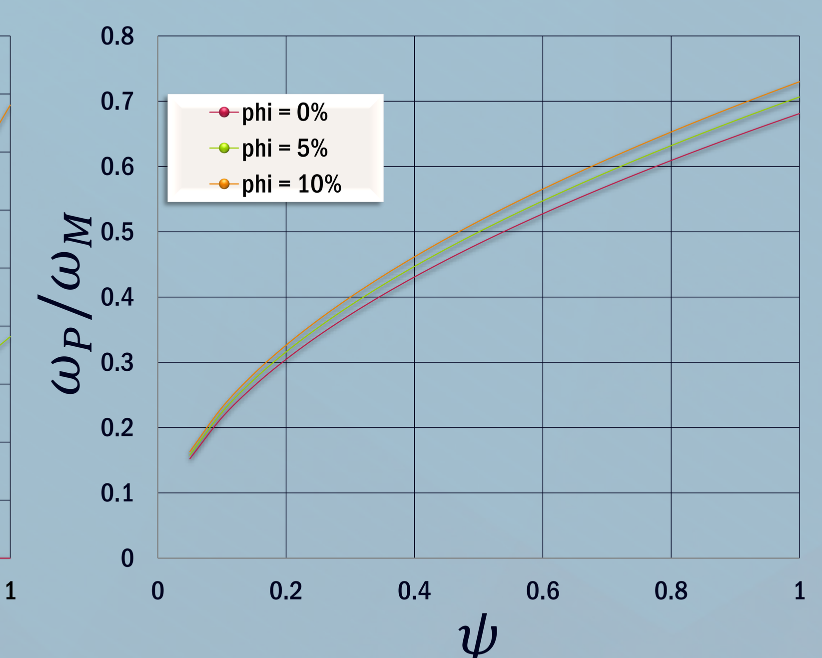
Transitional flow: $a = 42.9 + 23.9n$, $v = 1 - 0.33n$

Also, the following ratio constraint must be applied to the scaled system:

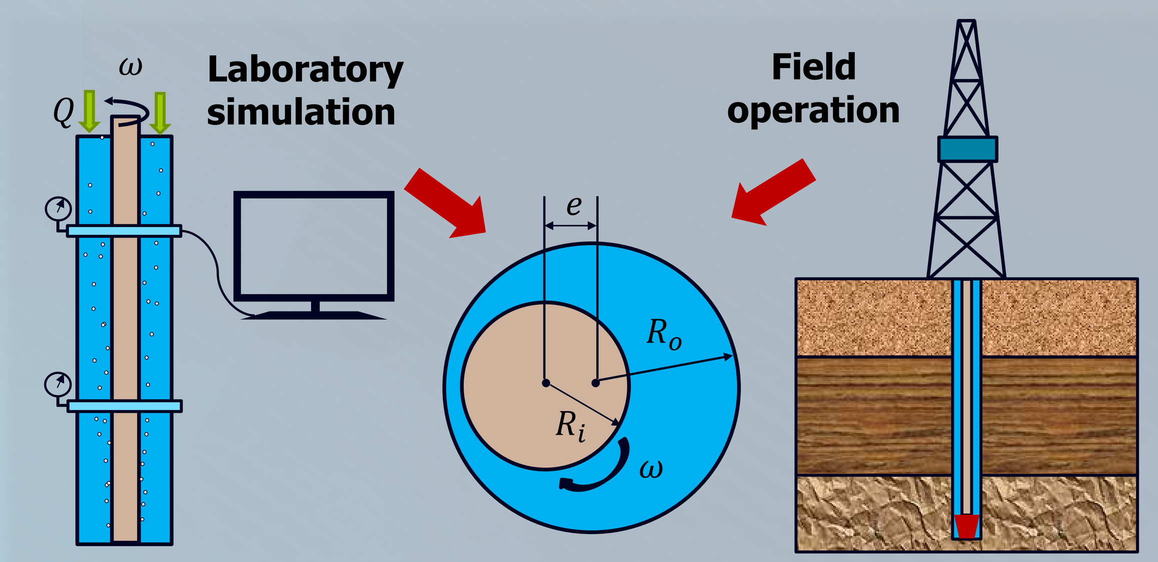
$$\frac{K_{fl,M}}{K_{fl,P}} = \left\{\frac{\rho_{fl,M}\, \psi^n \left[\frac{\psi^{1+nv}\rho_{fl,P}^{1-v}\,\rho_{fl,M}^{-1+v}(\rho_{fl,M} - \rho_{s,M})}{\rho_{fl,P} - \rho_{s,P}}\right]^{\frac{2-n}{2+v(-2+n)}}}{\rho_{fl,P}}\right\}^{\frac{2+v(n-2)}{2}}$$







Left: Laboratory fluid rate | Middle: Laboratory solid flow rate
Right: Drilling inner pipe to Laboratory inner pipe rotation speeds.



Schematics of fluid flow in wellbore and laboratory mini-flow loop.

Conclusions

- Key design parameters for a benchtop rig hydraulics similitude device are estimated. These parameters include pump flow rate, particles injection rate as well as the inner pipe rotation speed in the laboratory setup.
- After applying the geometric, dynamic, kinematic and rheological similarities, a direct relationship between frictional pressure drop along the piping of laboratory flow-loop device to the wellbore wall is obtained.
- Power law single phase fluid contained 9 variables, 3 dimensions, and 6 dimensionless groups. Incorporation of cuttings in the similitude introduced 3 more variables, upon which the mixture dimensionless groups became 9.
- Involvement of solid particles increased the complexity of the mixture; it imposed a new constraint on the rheology of fluid being used for the scaling relations.

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